Redundancy and Resilience in Distributed Optimization

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Optimization

- Finding *the least cost* solution

- Find $\arg\min f(x) \qquad x \in X$

Optimization

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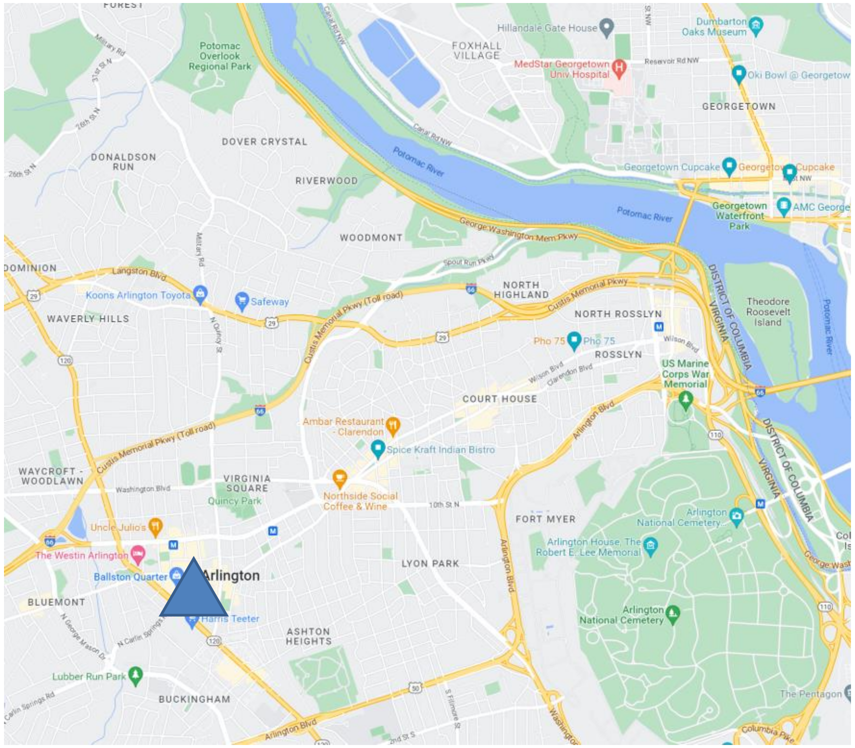
Optimization

- Finding *the least cost* solution

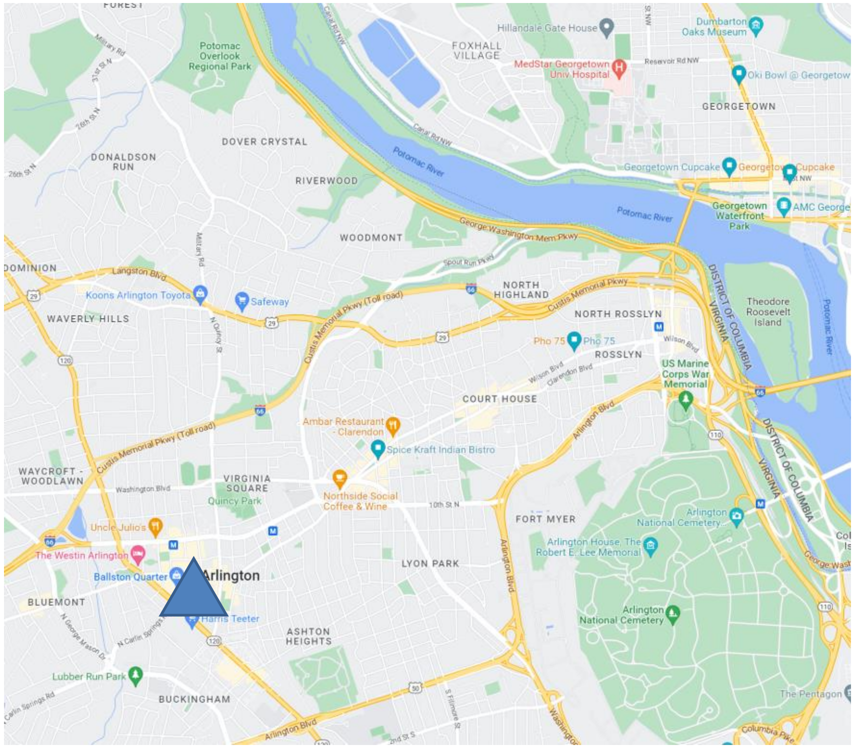
- Find $\arg\min f(x) \qquad x \in X$

- Many applications: ML, linear programming ...

Optimization

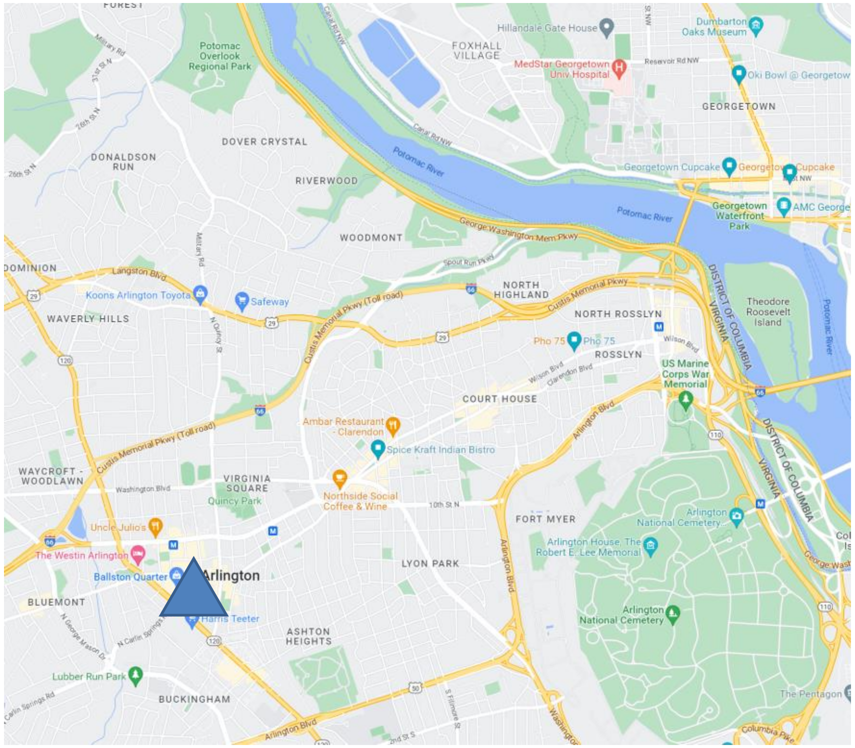


Optimization



Where cost is *minimum*

Optimization



Cost
Money, fuel, energy…
or adding them together

Where cost is *minimum*

Distributed optimization

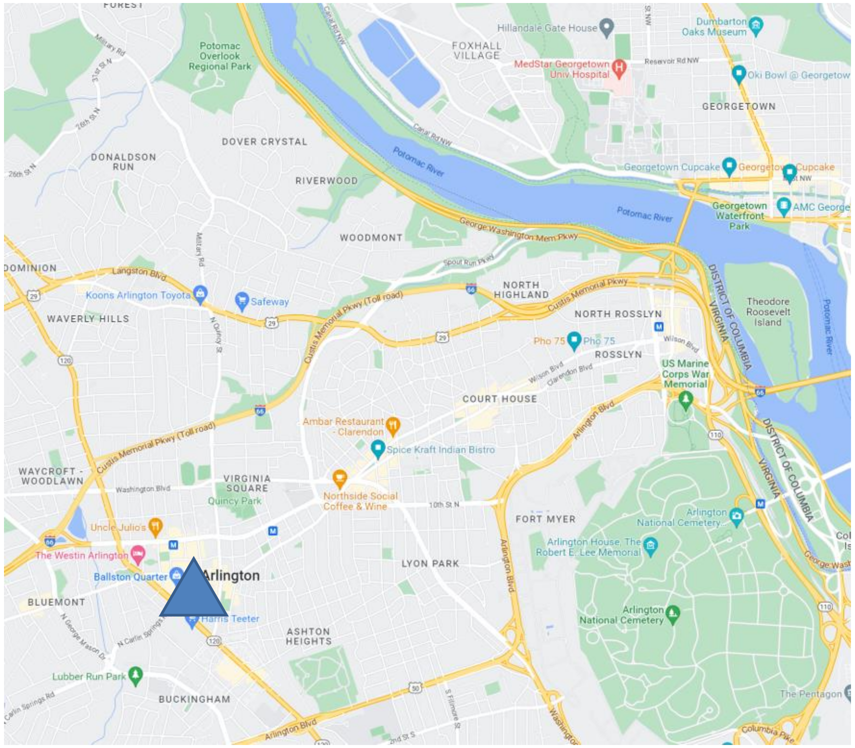
- n agents, each agent i has $f_i(x)$

- Find $\arg\min \sum_i f_i(x)$

Distributed optimization

- n agent, each agent i has $f_i(x)$
- Find $\arg\min \sum_i f_i(x)$
- Without sending the whole f_i's

Distributed optimization



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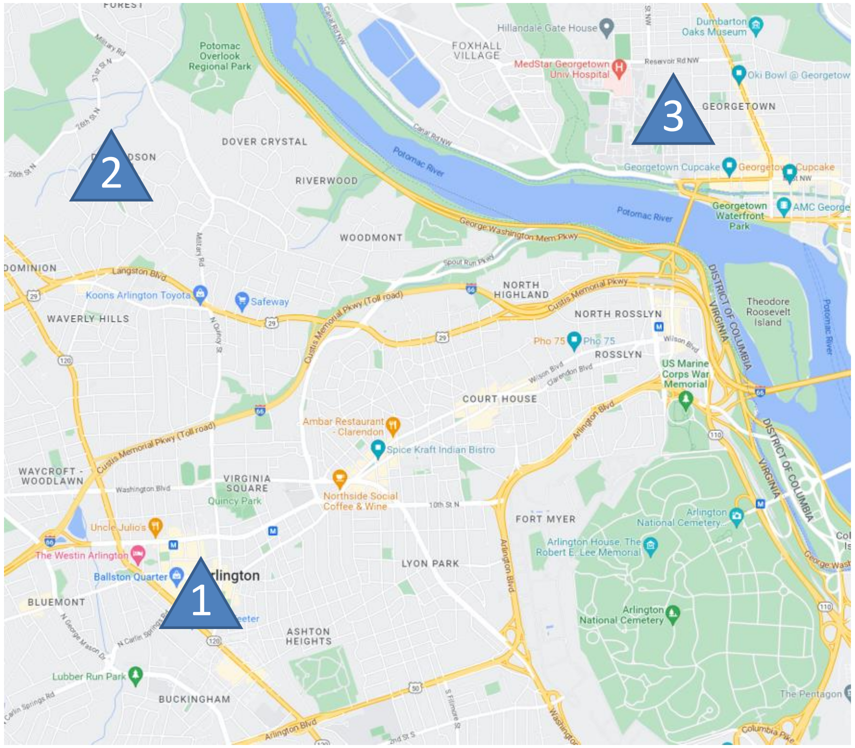
Distributed optimization



Where *aggregate* cost is minimum

Application to machine learning

- Each agent has a local dataset

Application to machine learning

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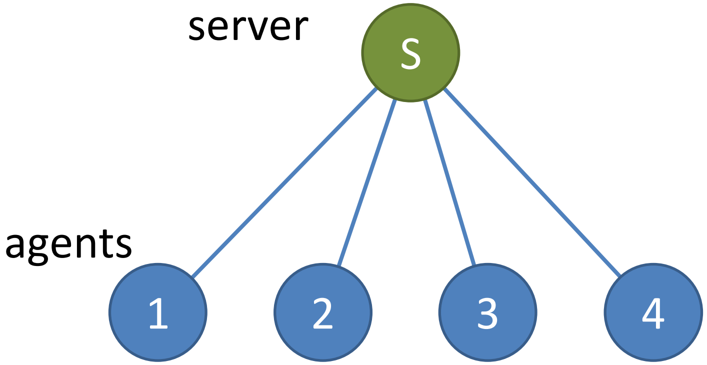
- **Agent's cost function:** Loss corresponding to its dataset

Application to machine learning

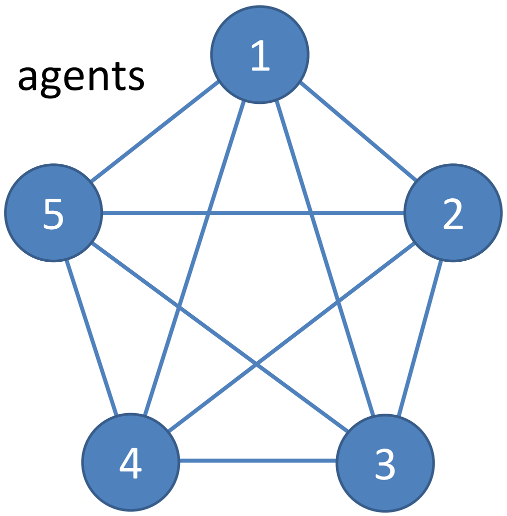
- Each agent has a local dataset
- **Agent's cost function:** Loss corresponding to its dataset
- **Goal:** Minimize the aggregate cost → train model

Distributed optimization

- Two common architectures



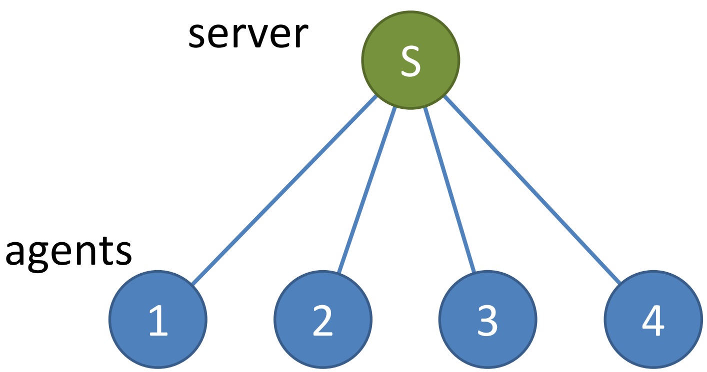
server-based



decentralized

Distributed optimization

- Two common architectures



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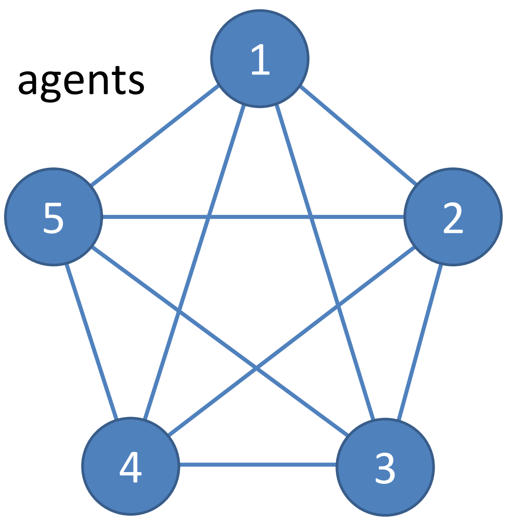
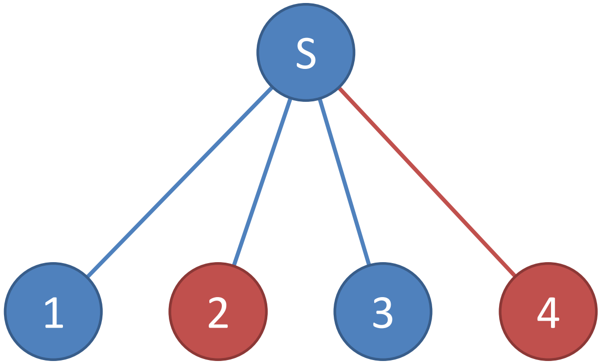


decentralized

- We focus on **server**-based architecture

Resilient distributed optimization

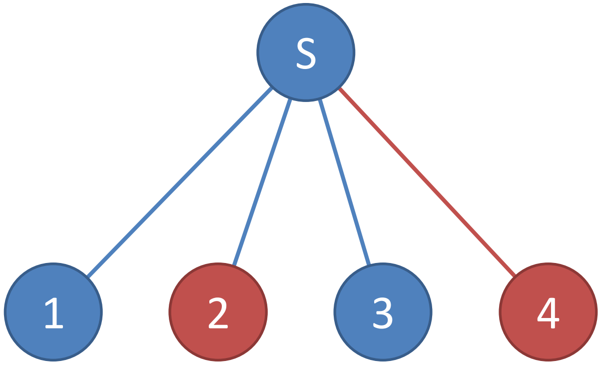
- Issues happen in practice
  - Faulty agents
  - Slow agents (stragglers)



Faulty agents

Fault models

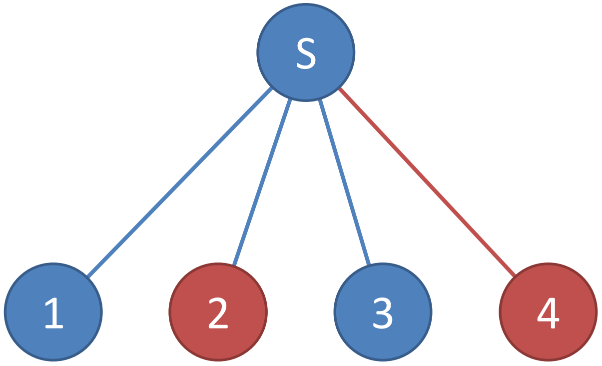
- Crash failures
- Byzantine model



Fault models

- Crash failures

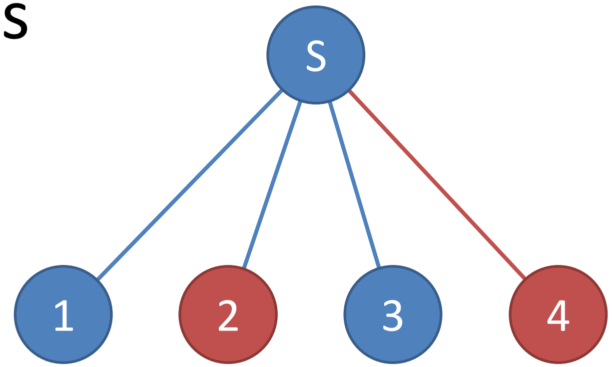
- **Byzantine model**
  - Software errors
  - Adversarial attacks



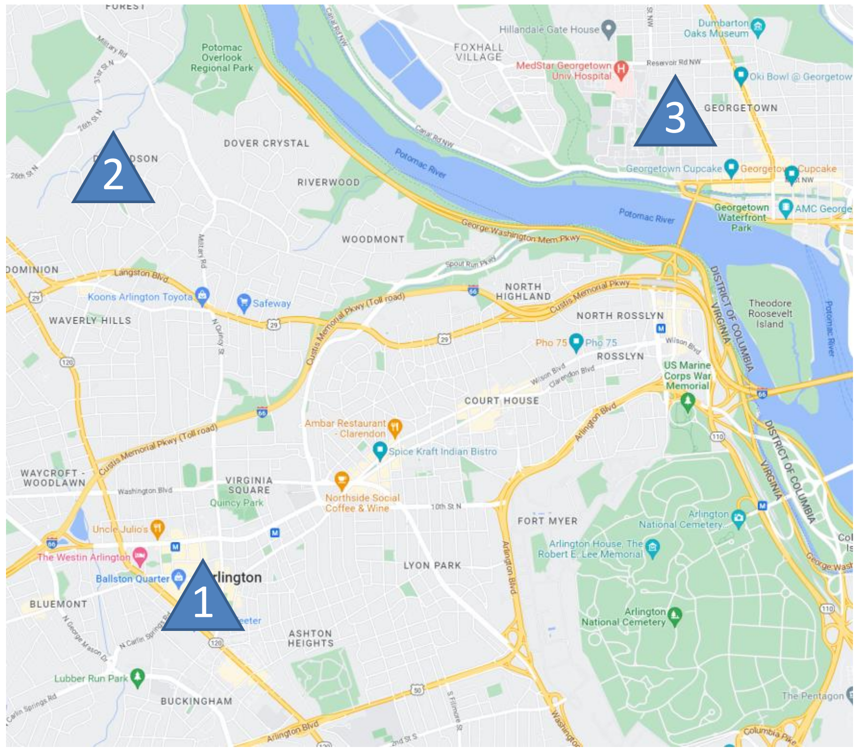
Byzantine model

- Make no assumption on agent behavior
- Cap the number of faulty agents

An algorithm that tolerates f faulty agents



Impact of faulty agents



Where *aggregate* cost is minimum

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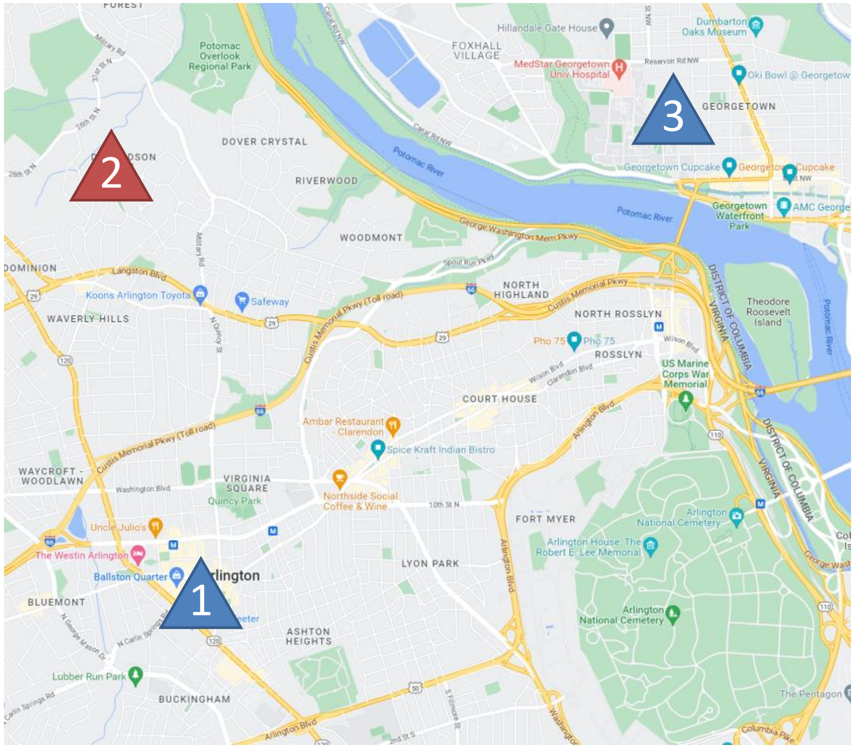
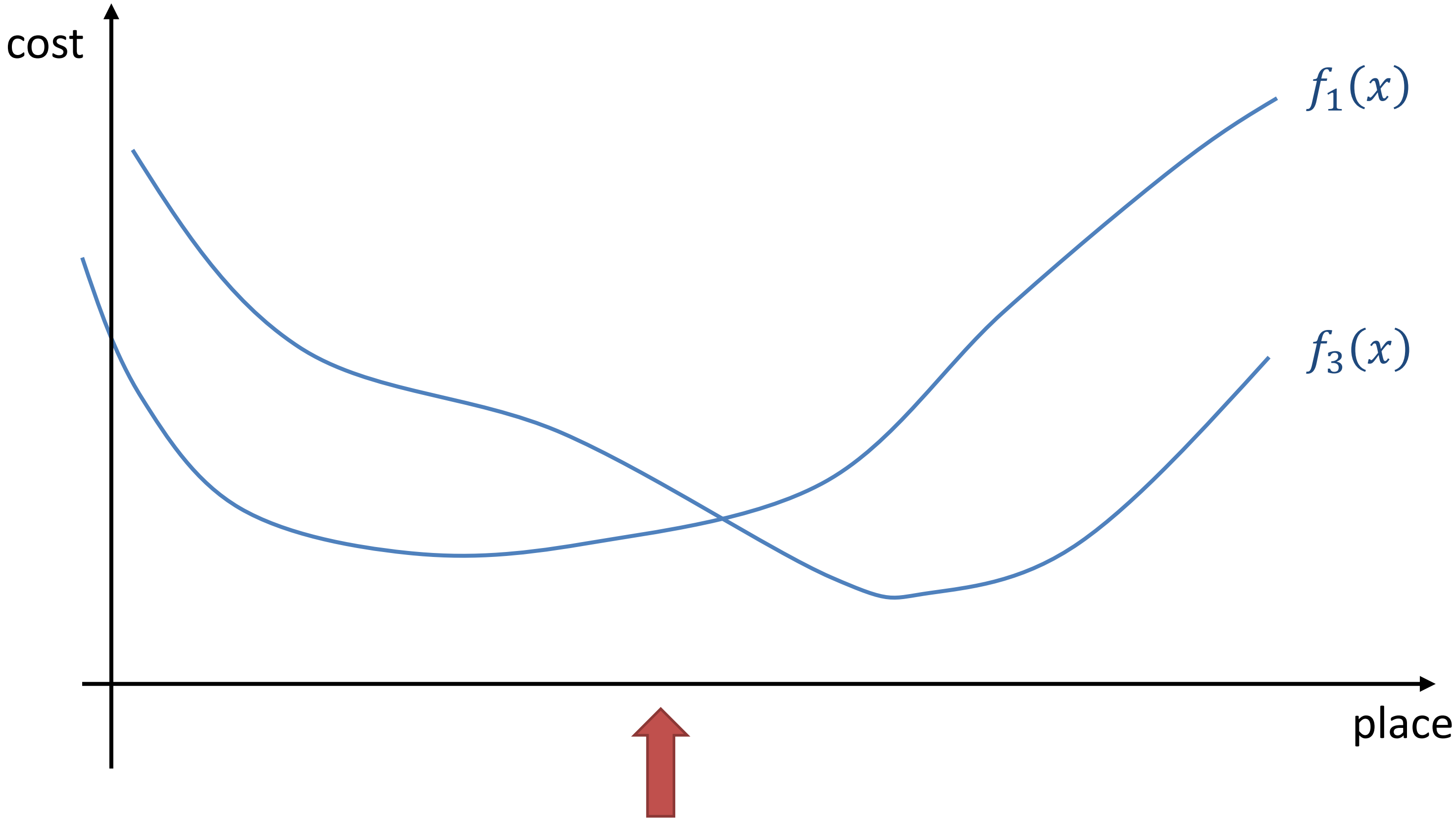


“I only want us to meet there”

Impact of faulty agents



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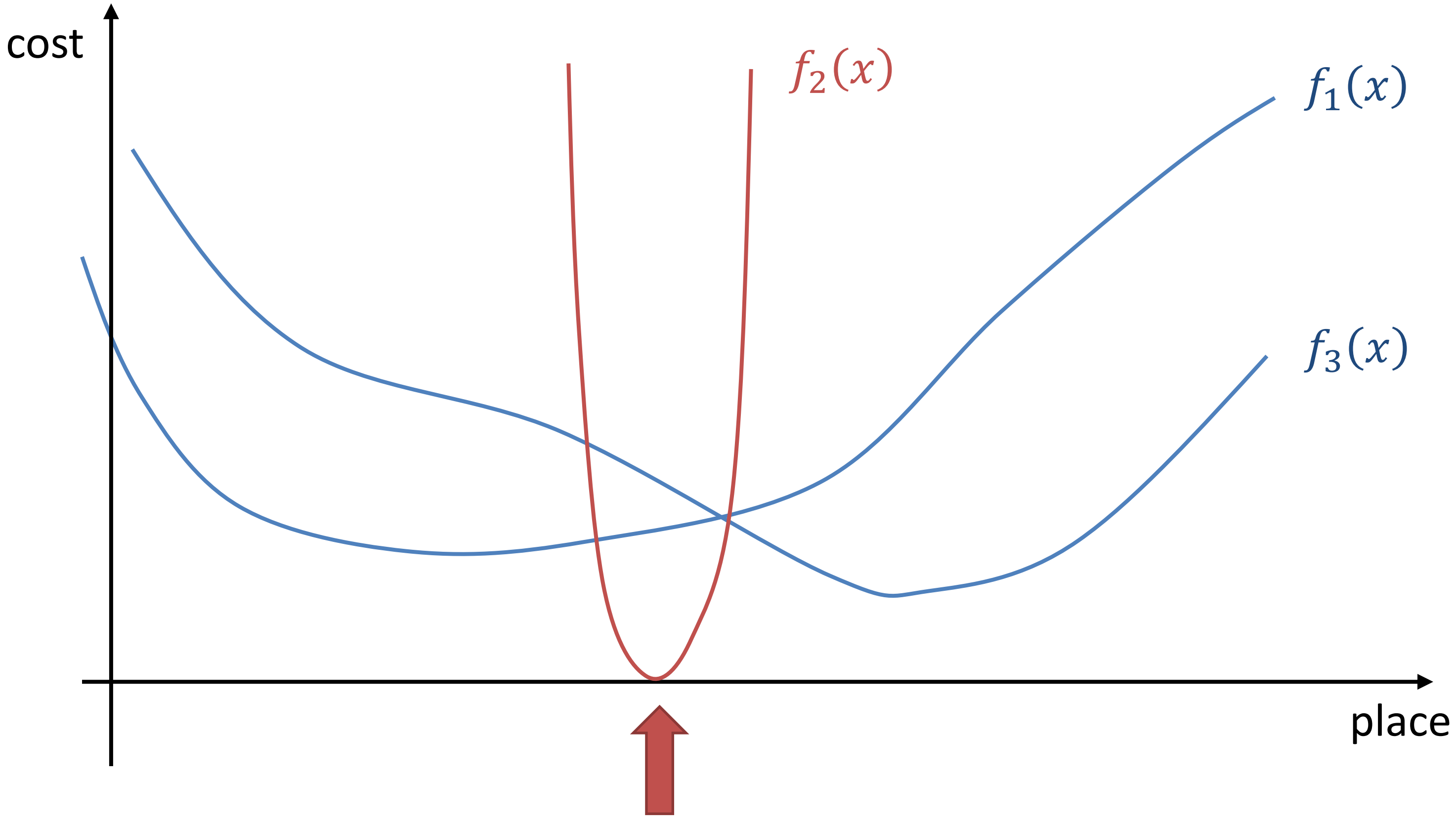

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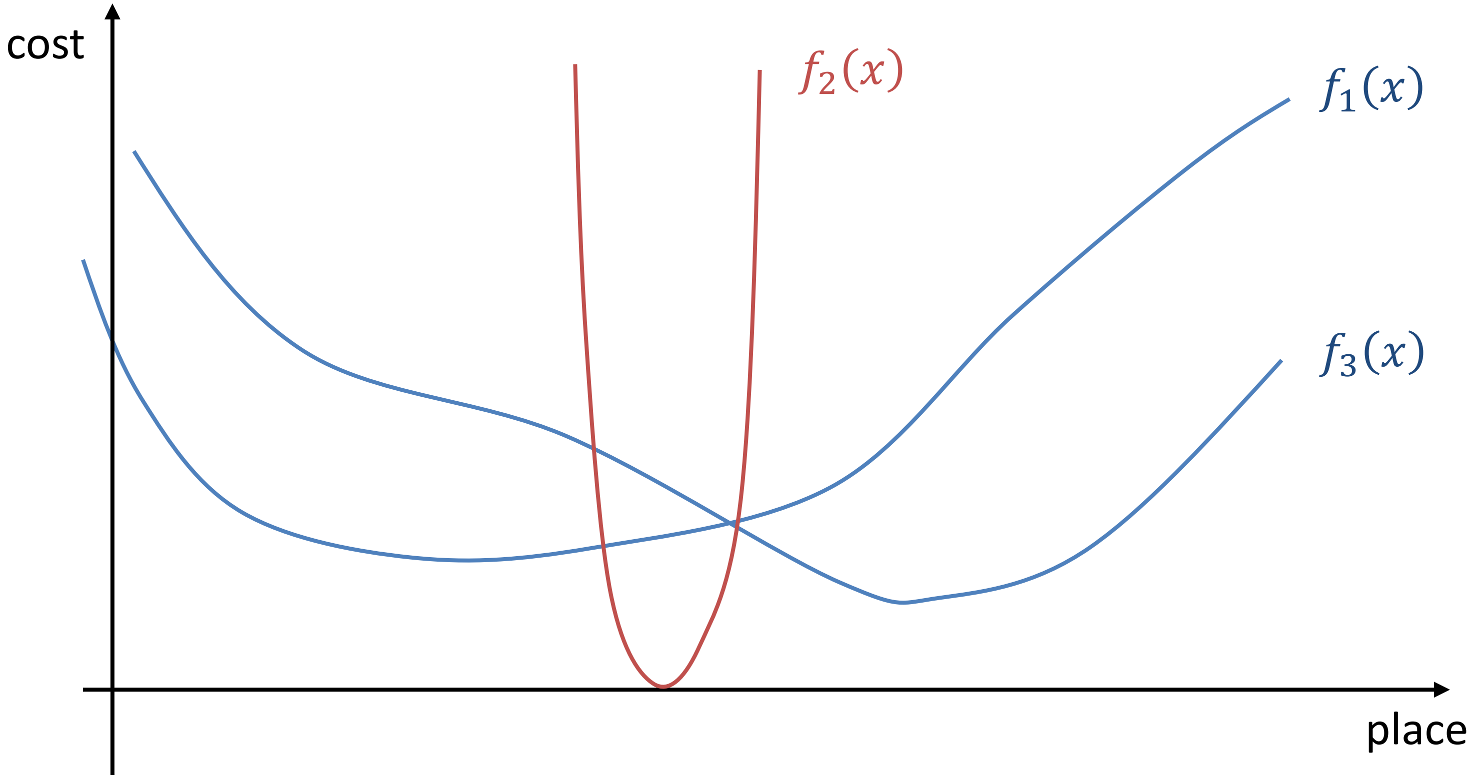
Impact of faulty agents



One faulty agent can disrupt the system

Impact of faulty agents

- Solving $\arg\min \sum_i f_i(x)$ is not useful
  - Faulty agents can send adversarial information

Impact of faulty agents

- Solving $\arg\min \sum_i f_i(x)$ is not useful
  - Faulty agents can send adversarial information

- What should be the goal of fault-tolerant optimization?

Fault-tolerance goal

- Minimize cost over only honest agents

- Find $\arg\min \sum_{\text{honest } i} f_i(x)$

$$\arg\min \sum_{\text{honest } i} f_i(x)$$

- Not achievable in general
- When can we achieve it exactly?

  ➔ Exact fault-tolerance
- When can we achieve it approximately?

  ➔ Approximate fault-tolerance

Exact fault-tolerance

Exact fault-tolerance

- Output $\arg\min \sum_{\mathrm{honest}\ i} f_i(x)$

Exact fault-tolerance

- Output $\arg\min \sum_{\text{honest } i} f_i(x)$

- Impossible without **redundancy** in cost functions

Redundancy

- Correlations among cost functions

- Common in practice
  - Observing the same object
  - Similarity between datasets
  - Correlation among data points

Exact fault-tolerance

- Output $\arg\min \sum_{\text{honest } i} f_i(x)$

- What type of redundancy required for exact fault-tolerance?

[Gupta & Vaidya, 2020]

$2f$-redundancy

[Gupta & Vaidya, 2020]

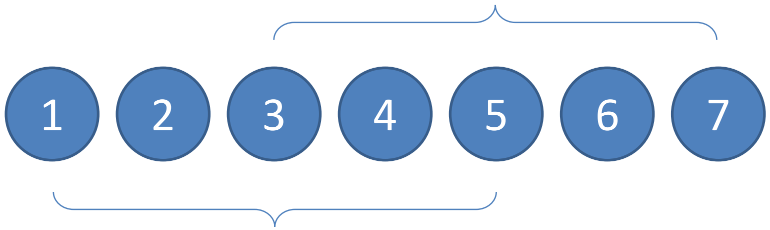
$\boxed{\text{Aggregate of every } n - 2f \text{ functions has the same minimum set}}$

$2f$-redundancy

[Gupta & Vaidya, 2020]

Aggregate of every $n - 2f$ functions has the same minimum set

$$x_2 = \arg\min \textstyle\sum_{i=3}^{7} f_i(x)$$



$$x_1 = \arg\min \textstyle\sum_{i=1}^{5} f_i(x)$$

$$n = 7$$
$$f = 1$$

$$x_1 = x_2 = \arg\min \textstyle\sum (5 \text{ functions})$$

$2f$-redundancy [Gupta & Vaidya, 2020]

Aggregate of every $n - 2f$ functions has the same minimum set

$2f$-redundancy $\Longleftrightarrow$ Exact fault-tolerance

$2f$-redundancy

- Strong condition, but not impossible
  - Example: replicated datasets

$2f$-redundancy

- Strong condition, but not impossible
  - Example: replicated datasets

- Difficult to satisfy in general

- Can we define a weaker goal?

Approximate fault-tolerance

Approximate fault-tolerance

- Need a measure for approximation

- We define (f, ϵ)-resilience

(f, ϵ)-resilience

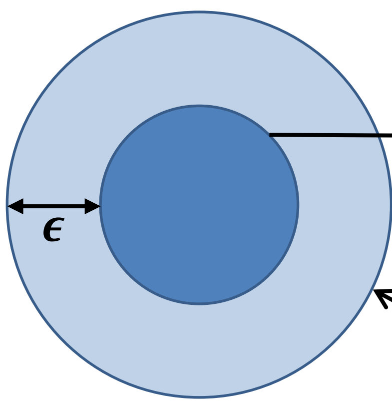
Algorithm output within ϵ of minimum for aggregate of every $n - f$ honest functions, in presence of f Byzantine agents

$$\text{distance}\left(\text{output},\ \ \arg\min \sum_{\substack{\text{honest}\\ n-f}} f_i(x)\right) \leq \epsilon$$

(f, ϵ)-resilience

$$\text{distance}\left(\text{output}, \ \arg\min \sum_{\substack{\text{honest} \\ n-f}} f_i(x)\right) \leq \epsilon$$

ϵ – error margin of the algorithm



$$x_{\mathcal{H}} = \arg\min \sum_{i \in \mathcal{H}} f_i(x)$$

Output within

$\mathcal{H}$ is set of $n - f$ honest agents

Approximate fault-tolerance

- (f, ϵ)-resilience describes an algorithm
- **Goal:** Achieve (f, ϵ)-resilience
- This requires adequate redundancy in cost functions

$(2f, \epsilon)$-redundancy

Aggregate of any $n - f$ cost functions and its subset of size $\geq n - 2f$ have minimizers within ϵ of each other

$$\text{distance}\left(\arg\min \sum_{\substack{\text{honest} \\ n-f}} f_i(x), \arg\min \sum_{\substack{\geq n-2f \\ subset}} f_i(x)\right) \leq \epsilon$$

$(2f, \epsilon)$-redundancy

$$\boxed{\text{distance}\left(\arg\min \sum_{\substack{\text{honest}\\ n-f}} f_i(x)\,, \arg\min \sum_{\substack{\geq n-2f\\ subset}} f_i(x)\right) \leq \epsilon}$$

ϵ – how redundant the costs functions are

$(2f, \epsilon)$-redundancy

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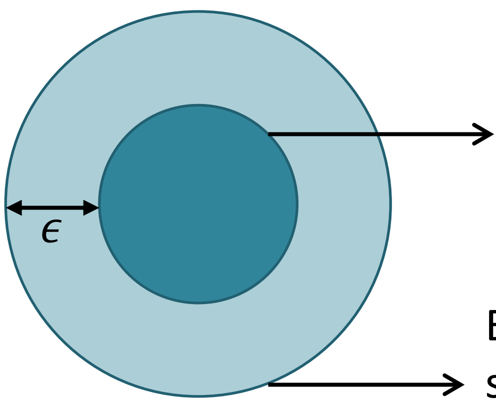

$$x_{\mathcal{H}} = \arg\min \sum_{i \in \mathcal{H}} f_i(x)$$

Every $\geq n - 2f$
subset minimizes
within this range

$(2f, \epsilon)$-redundancy

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$$\text{distance}\left(\arg\min \sum_{\substack{\text{honest} \\ n-f}} f_i(x), \arg\min \sum_{\substack{\geq n-2f \\ subset}} f_i(x)\right) \leq \epsilon$$

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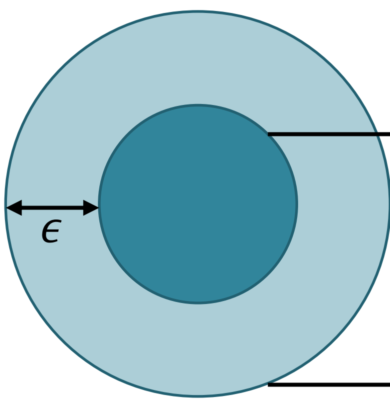
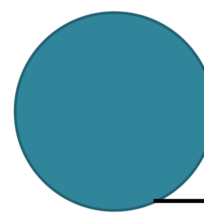


$x_{\mathcal{H}} = \arg\min \sum_{i \in \mathcal{H}} f_i(x)$

Every $\geq n - 2f$ subset minimizes within this range

$(2f, \epsilon)$-redundancy



Both $x_{\mathcal{H}}$ and minimizers of $n - 2f$ subsets

$2f$-redundancy
$(\epsilon = 0)$

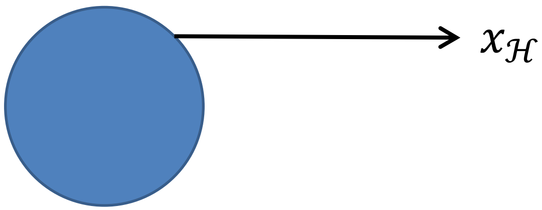
$(2f, \epsilon)$-redundancy

- $(2f, \epsilon)$-redundancy describes cost functions
- Can we achieve resilience with $(2f, \epsilon)$-redundancy?

Theoretical results

Necessity

(f, ϵ)-resilience can be achieved only if cost functions satisfy $(2f, \epsilon)$-redundancy



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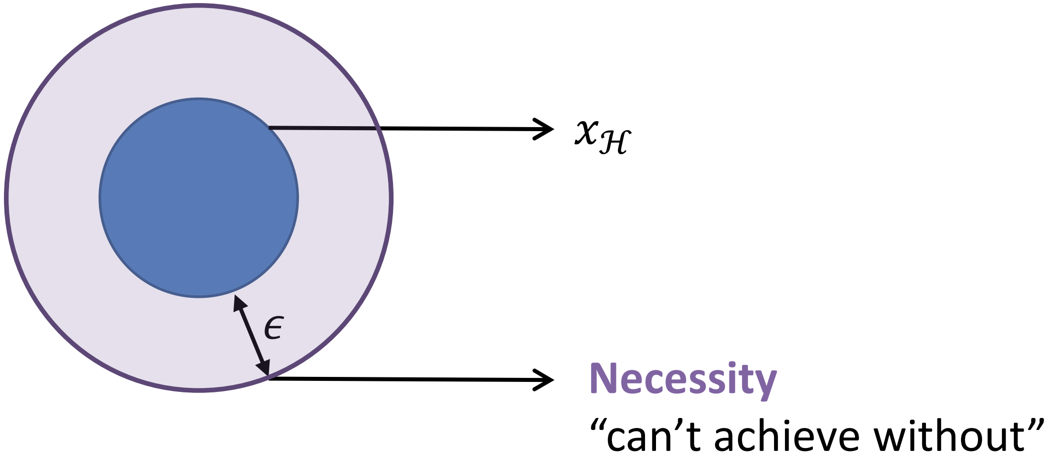
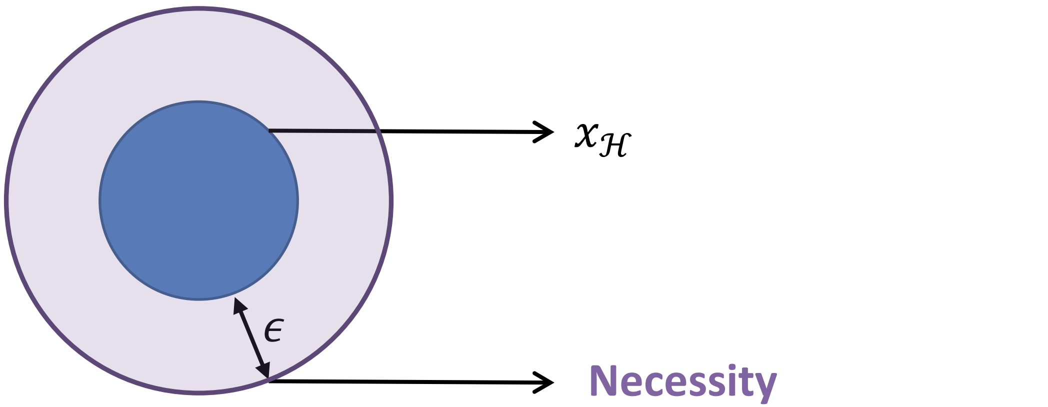


Necessity

(f, ϵ)-resilience can be achieved only if cost functions satisfy $(2f, \epsilon)$-redundancy

Sufficiency

If the cost functions satisfy $(2f, \epsilon)$-redundancy, $(f, 2\epsilon)$-resilience can be achieved



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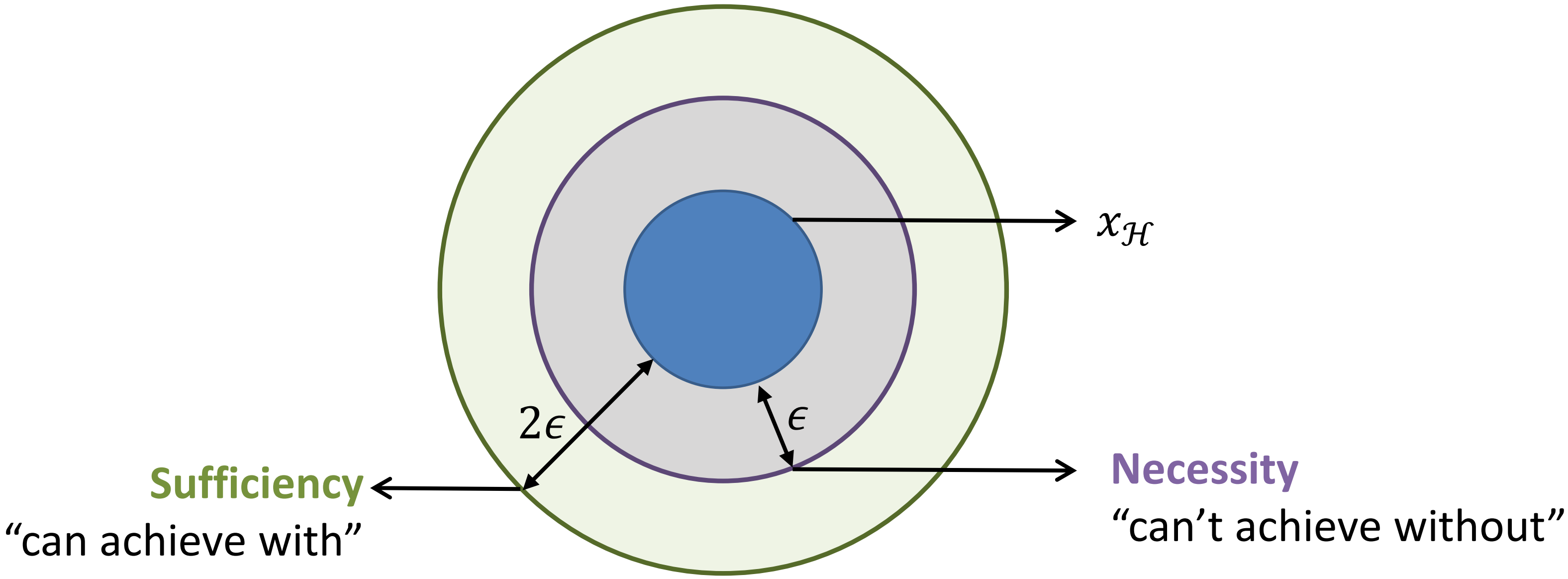


Approximate fault-tolerance

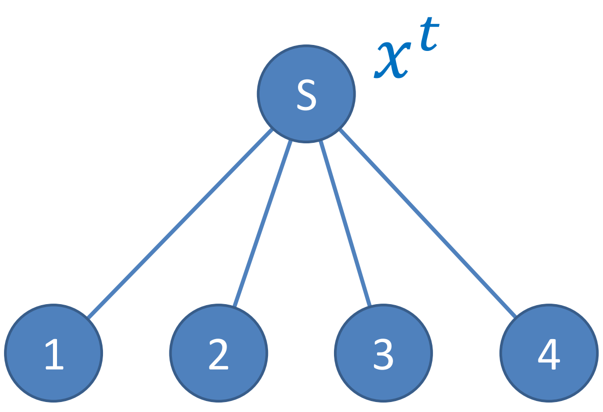
- **Result:** $\left(f, O(\epsilon)\right)$-resilience is achievable with $(2f, \epsilon)$-redundancy, and not achievable without *(In theory)*

- Is there any practical solution?

Fault-tolerant algorithm

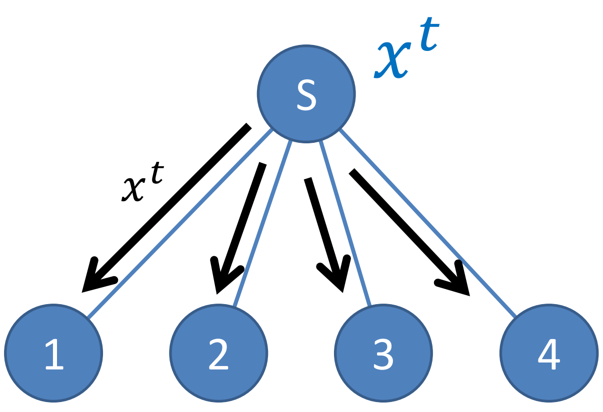
Distributed gradient descent

- Server maintains estimate x^t
- In each iteration t



Distributed gradient descent

- Server maintains estimate x^t
- In each iteration t
  - Server broadcasts current estimate x^t



Distributed gradient descent

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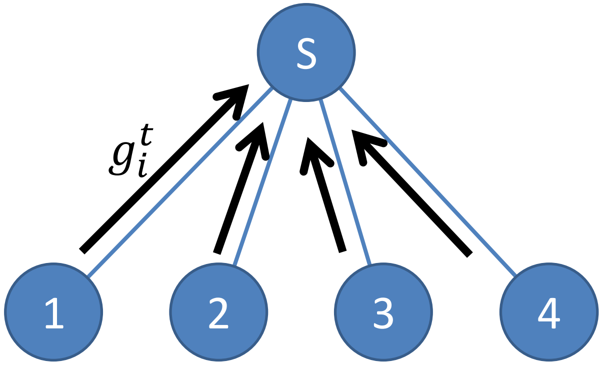
    Agents send back gradients g_i^t

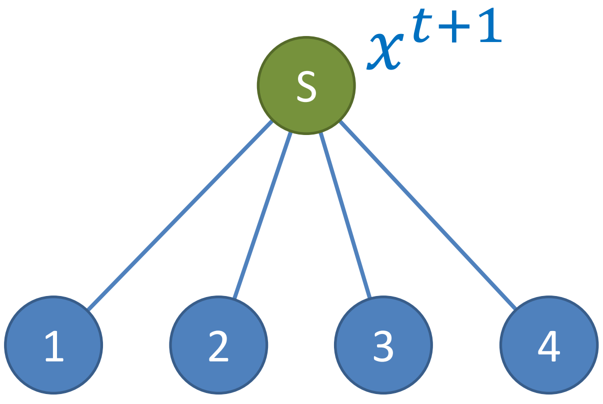


Distributed gradient descent

- Server maintains estimate x^t
- In each iteration t
  - Server broadcasts current estimate x^t

    Agents send back gradients g_i^t
  - Server collects the gradients and makes an update by

$$x^{t+1} = x^t - \eta \cdot \sum_i g_i^t$$



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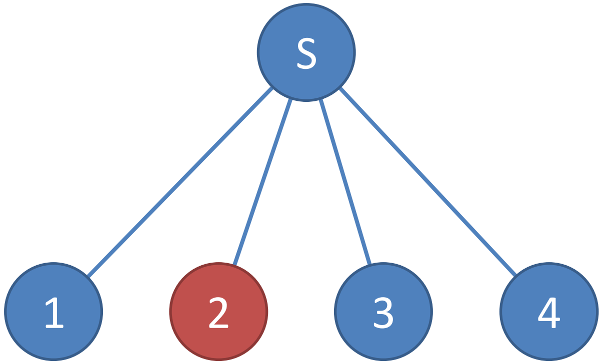
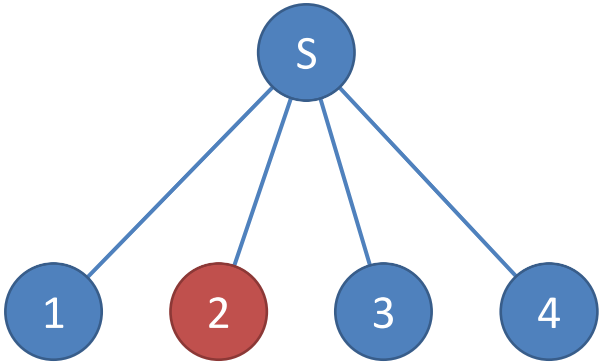


- Fails with faults

Fault-tolerant algorithm

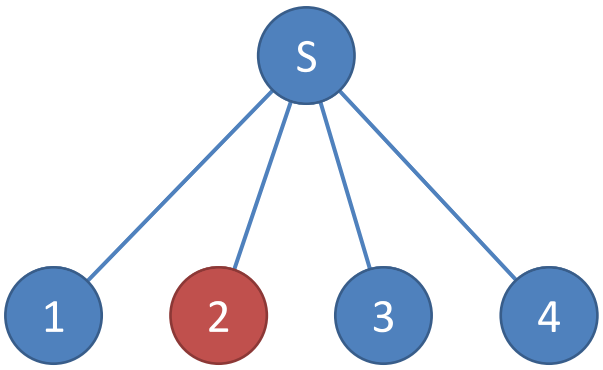
- Distributed gradients descent with filters
- In each iteration t



Fault-tolerant algorithm

- Distributed gradients descent with filters
- In each iteration t
  - Server broadcasts current estimate x^t

    Agents send back gradients or faulty vectors g_i^t

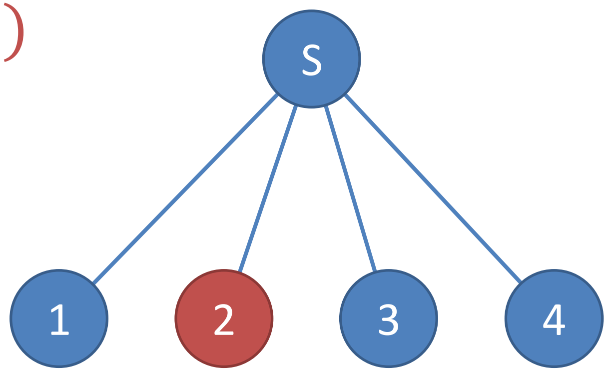


Fault-tolerant algorithm

- Distributed gradients descent with filters
- In each iteration t
  - Server broadcasts current estimate x^t

    Agents send back gradients or faulty vectors g_i^t
  - Server collects and filters the vectors, and makes an update

$$x^{t+1} = x^t - \eta \cdot \mathrm{GradFil}(g_i^t)$$



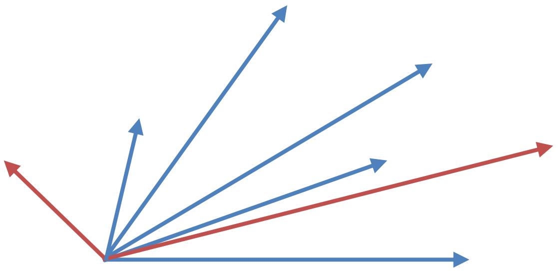
Gradient filters

- Also called gradient aggregation rules
- Mitigate faulty vectors
- Many designs
  - Krum [Blanchard et al., 2017]
  - Coordinate-wise methods [Yin et al., 2018]
  - Comparative gradient elimination [Gupta & Vaidya, 2019]
  - Geometric median, median of means, … [Chen et al., 2017]

Gradient filters

- Comparative gradient elimination

$$\boldsymbol{n = 7}$$
$$\boldsymbol{f = 2}$$



Gradient filters

$\boldsymbol{n = 7}$
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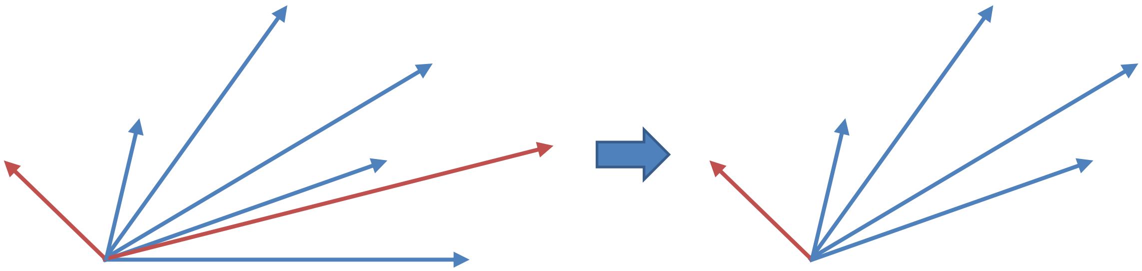
- Comparative gradient elimination



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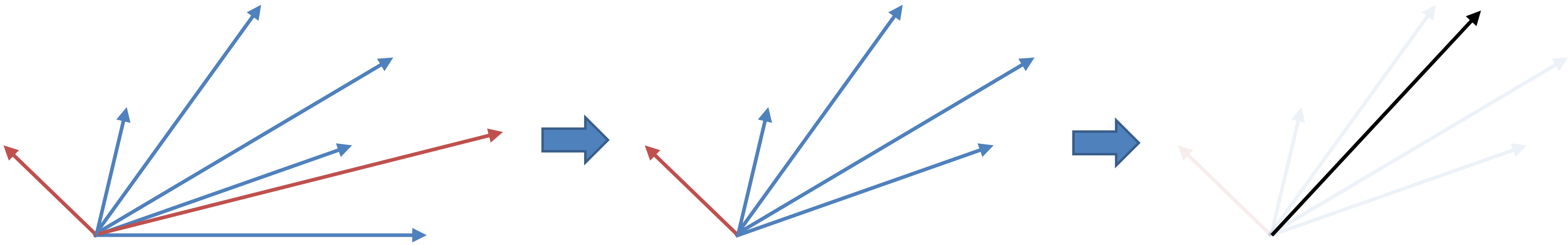

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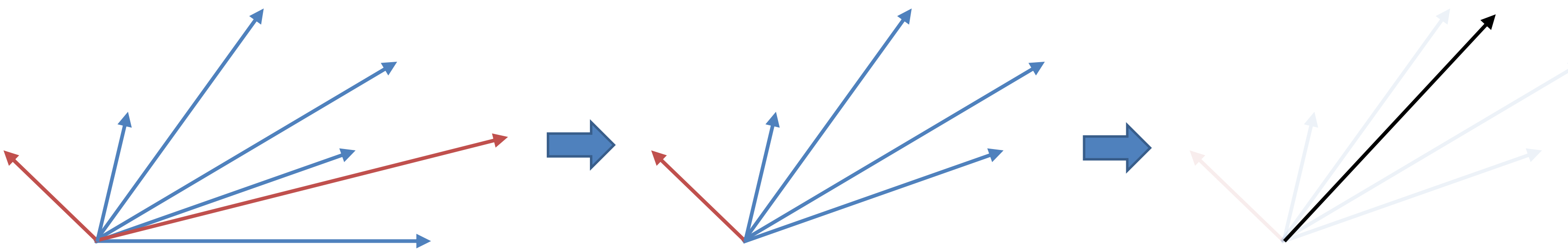


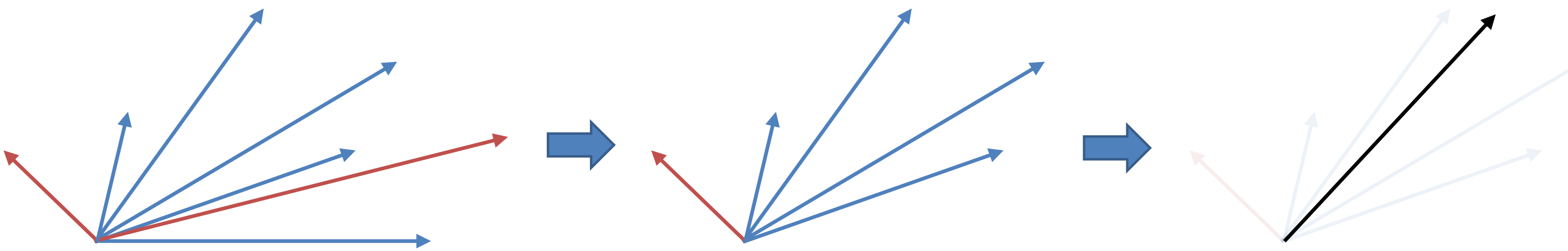
- Coordinate-wise trimmed mean

			⋮			
1	4	5	2	3	5	6
			⋮			

Gradient filters

- Comparative gradient elimination

$$n = 7$$
$$f = 2$$



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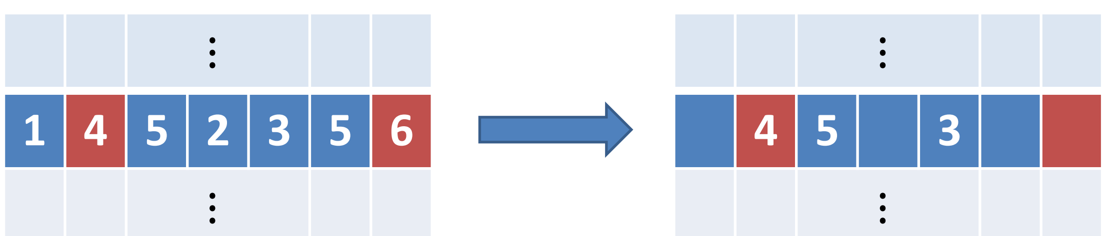

Gradient filters

$n = 7$
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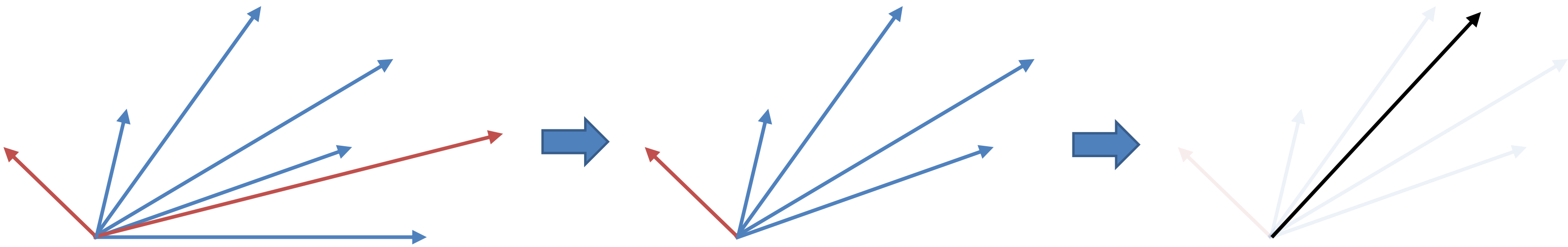

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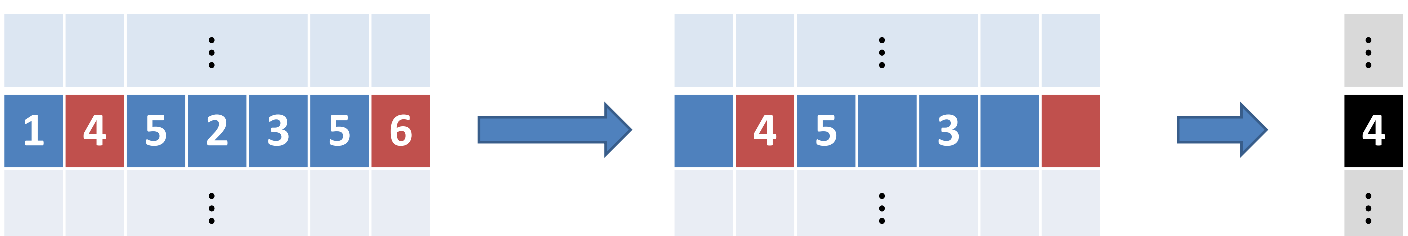


Performance

With $(2f, \epsilon)$-redundancy, the algorithm is $\left(f, \mathcal{O}(\epsilon)\right)$-resilient

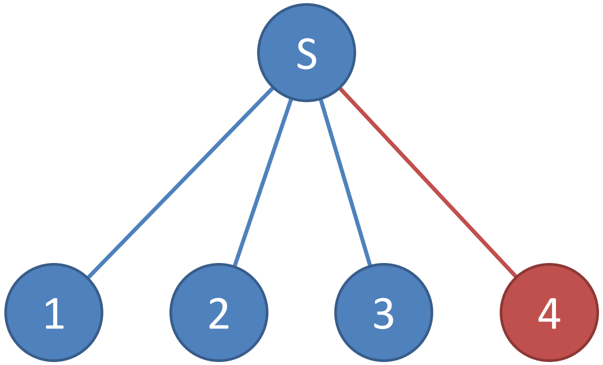
- Error margin decided by gradient filter
- Gradient elimination requires $f \leq n/3$
- Trimmed mean's margin depends on parameter vector size

• Trade-off between complexity and error margin

Stragglers + Byzantine agents

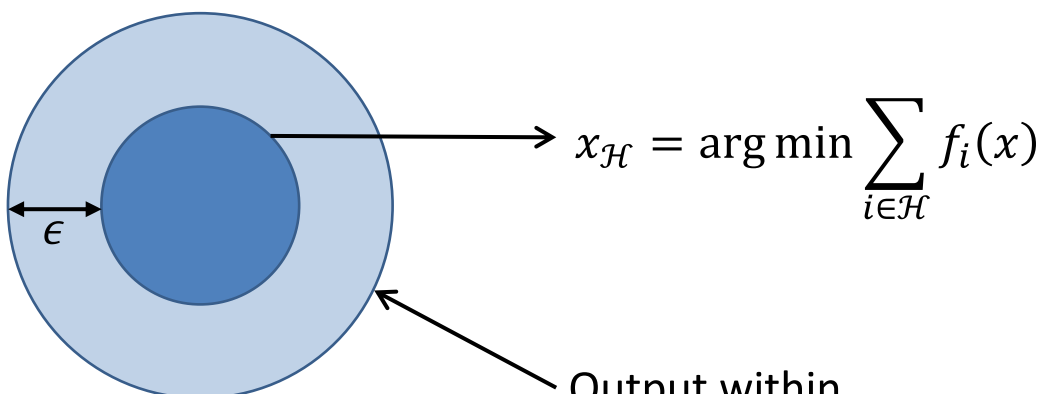
Stragglers

- Stragglers are slow agents
- Stragglers delay synchronous algorithm
- Solution
  - Don’t wait for slow agents
  - Exploit redundancy



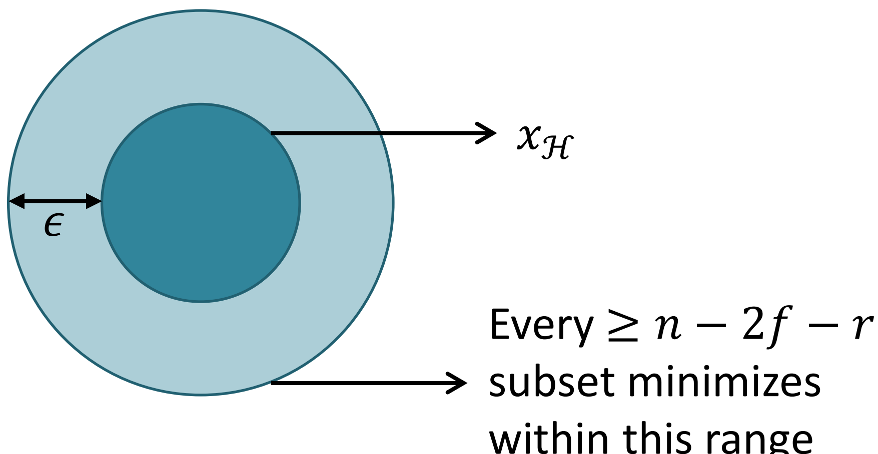
Algorithm: $(f, r; \epsilon)$-resilient

Algorithm output within ϵ of minimum for aggregate of every $n - f$ honest functions in presence of f Byzantine agents and r stragglers



Costs: $(f, r; \epsilon)$-redundancy

Aggregate of any $n - f$ cost functions and its subset of size $\geq n - 2f - r$ have minimizers within ϵ of each other



$(f, r; \epsilon)$-redundancy

Resilient distributed optimization

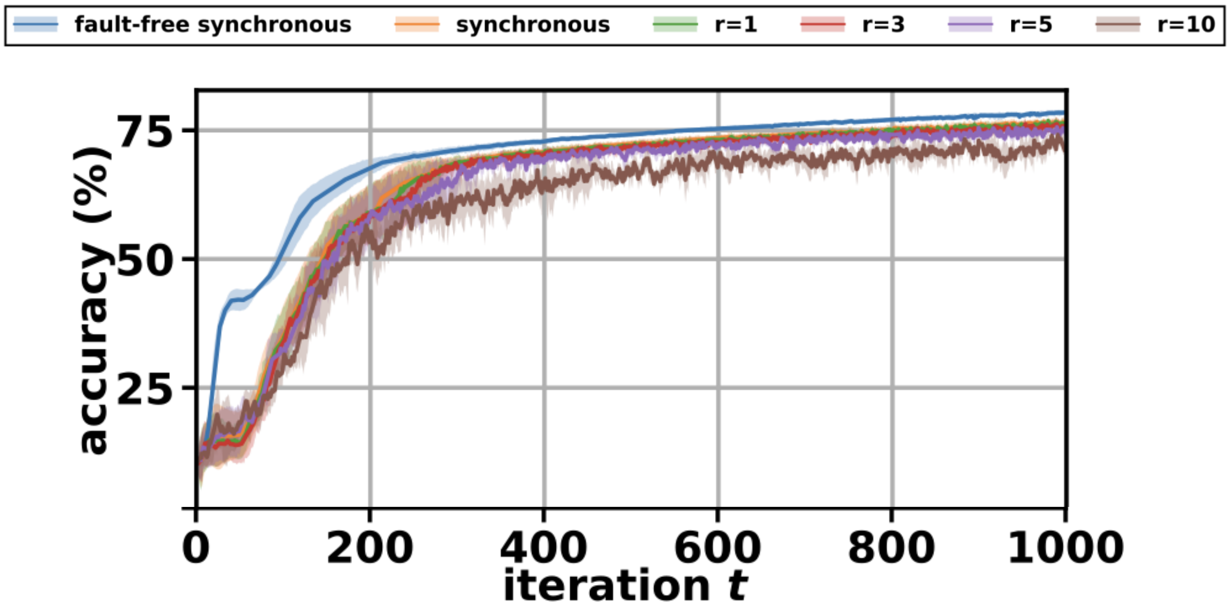
- Can show similar necessity for $(f, r; \epsilon)$-redundancy

- Can show $\left(f, r; \mathcal{O}(\epsilon)\right)$-resilience for DGD + filter

Experiments

f: faulty agents
r: stragglers

- MNIST on LeNet – Label-flipping faults
- $n = 20, f = 3$, various r's



Summary

- Defined (f, ϵ)-resilience and $(2f, \epsilon)$-redundancy
- Obtained necessary and sufficient conditions
- Algorithm with gradient filters
- Extended to stragglers

Thank you

Thank you

Questions

Presented papers

- Nirupam Gupta and Nitin H. Vaidya. Fault-Tolerance in Distributed Optimization: The Case of Redundancy. (PODC 2020)
- Shuo Liu, Nirupam Gupta, and Nitin H. Vaidya. Approximate Byzantine Fault-Tolerance in Distributed Optimization. (PODC 2021)
- Shuo Liu, Nirupam Gupta, and Nitin H Vaidya. Impact of Redundancy on Resilience in Distributed Optimization and Learning. (ICDCN 2023)